MATH 141 Sample Final Exam Problems

Question 1 Evaluate the derivatives of the following functions:

a) $f(x) = \sin\sqrt{x^2+x+1}\cdot\cos\sqrt[3]{x}$

b) $f(x) = \dfrac{(x^2+1)}{\cos^3(3x+1)}$

c) $f(x) = x^3\tan(3x^2) + \sec(1/x)$

d) $f(x) = \displaystyle\int_{-3x}^{\sin x} \frac{1}{1+t^2}\,dt$

Question 2 Find the equation of the tangent line to the curve $y = f(x)$ at the point $(1,0)$ if $xy^2 + 5y = x^3 - \cos(xy)$

Question 3 Determine the following limits:

a) $\displaystyle\lim_{x\to -2} \frac{x^3+8}{x^2+x-2}$

b) $\displaystyle\lim_{x\to 0} \frac{\sin^2 8x}{\sin(8x^2)}$

c) $\displaystyle\lim_{x\to\infty} \sqrt{x^2+100x} - \sqrt{x^2+50x}$

d) $\displaystyle\lim_{x\to\infty} \frac{3+\sin(7x)}{x}$

Question 4 A trough is 10 feet long and its ends have the shape of isosceles triangles that are 3 feet across at the top and have a height of 1 foot. If the trough is being filled with water at a rate of 12 ft^3/min, how fast is the water level rising when the water is 6 inches deep?

Question 5 Find the area of the largest rectangle with sides parallel to the coordinate axes which can be inscribed in the ellipse $x^2/4 + y^2/9 = 1$.

Question 6 Consider the function $f(x) = \dfrac{3x}{x^2-4}$. Then $f'(x) = \dfrac{-3(x^2+4)}{(x^2-4)^2}$ and $f''(x) = \dfrac{6x(x^2+12)}{(x^2-4)^3}$. Find all vertical and horizontal asymptotes. Find all local maxima and minima. Describe the intervals where the function is concave up and down. Describe the intervals where the function is increasing and decreasing. Sketch graph.

Question 7 Use the definition of derivative and the identity $\sin(A+B) = \sin A\cos B + \cos A\sin B$ to find the derivative of $f(x) = \sin 5x$.

Question 8 Show that the equation $\dfrac{x^2(x-1)}{4+5x^2} = 1$ has at least one real solution.

Question 9 Let $f(x) = x^4 + Ax^2 + x$, where A is a constant. Are there values of A for which f has inflection points at both $x = 0$ and $x = 1$?

Question 10 Perform the required integrations:

a) $\displaystyle\int 6\sec^2 5x\;dx.$

b) $\displaystyle\int \frac{\cos x}{\sqrt{1+2\sin x}}\;dx$

c) $\displaystyle\int \frac{\cos\sqrt{x}}{5\sqrt{x}}\;dx$

d) $\displaystyle\int_0^1 x^5\sqrt[7]{(x^3+1)}\;dx$

Question 11

a) Use a right endpoint approximation to estimate the area under the curve $y = x + x^3$ over the interval $[0, 2]$ using 4 subintervals.

b) Find the exact area under the curve above by setting up a Riemann sum and taking an appropriate limit. You may need the formulas: $\sum_{i=1}^{n} i = \frac{1}{2}n(n+1)$, $\sum_{i=1}^{n} i^2 = \frac{1}{6}n(n+1)(2n+1)$, $\sum_{i=1}^{n} i^3 = \frac{1}{4}n^2(n+1)^2$

The topics covered on the final are Chapter 2, Chapter 3, Chapter 4 (except for sections 4.6, 4.8 & 4.9), Chapter 5. You will also be expected to know trigonometry stuff.